Exercise 5.1

APD receiver's sensitivity (defined as the minimum number of photons per bit for a given quality parameter Q) in the absence of dark current is found to be:

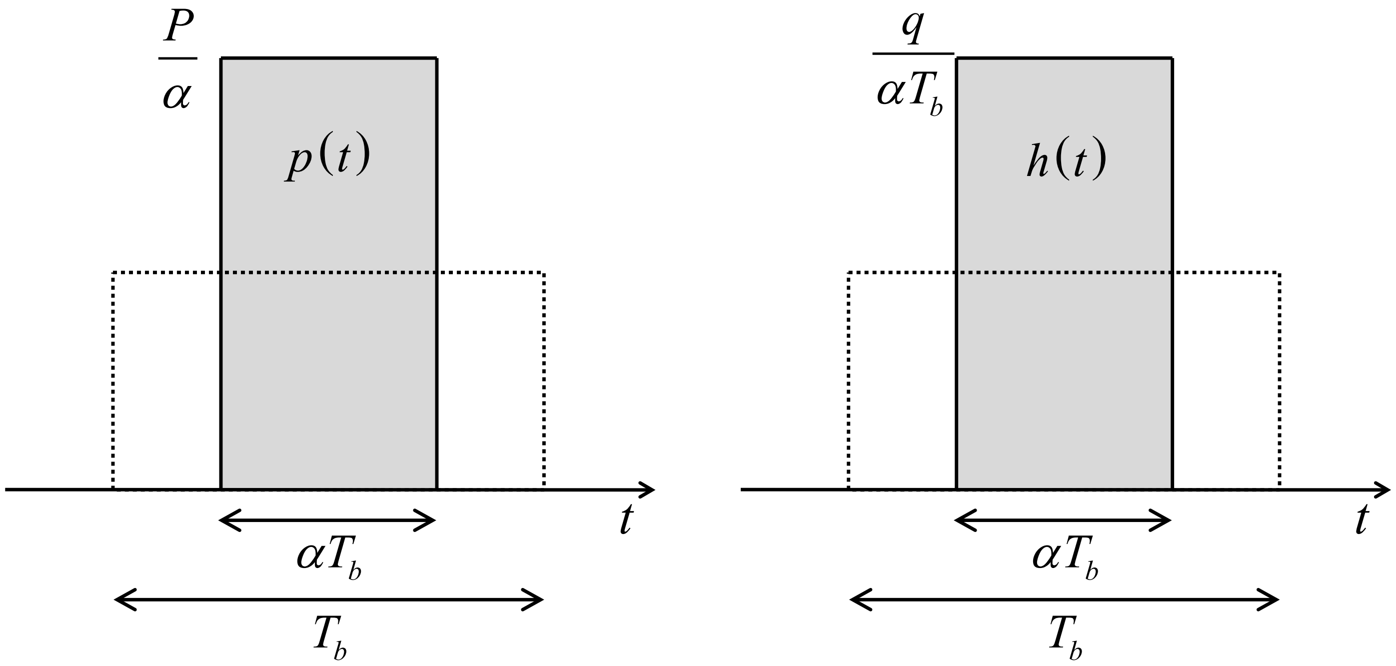
$$\langle n_a \rangle \geq \frac{Q}{\eta}\left(\frac{Q}{2}F + \frac{\sigma_p}{M}\right)$$

Where η, M, and F are the APD's quantum efficiency, multiplicative parameter, and noise factor, respectively. σ_p is the thermal noise standard deviation in number of electrons per bit.

1) Assuming a noise factor in the form $F(M) = M^x$, find the optimum multiplicative parameter (M).
2) Find the optimum receiver sensitivity.
3) Assuming and ideal quantum efficiency, a noise factor $F(M) = M$ and a thermal noise of $\sigma_p = 100$, find the optimum sensitivity for a quality parameter $Q = 6$ ($BER = 10^{-9}$). Compare it with the PIN photodetector under the same conditions of quantum efficiency and thermal noise.
4) Determine when the APD improves the sensitivity compared to a PIN. Take into account that the multiplicative parameter must be $M \geq 1$.

Exercise 5.2

Consider a $10\,Gb/s$ IM/DD fiber-optic system. The goal is to investigate the influence of the temporal width of both the optical pulses $p(t)$ and the receiver impulse response $h(t)$. For the sake of simplicity, they will be assumed with rectangular shape (see figure). It can be proven that, the best sensitivity is achieved when the duration of $h(t)$ matches the one of $p(t)$.



For a fair comparison, the pulse's energy must be kept constant $E_p = P \cdot T_b$, where T_b is the bit period. The receiver's impulse response is normalized so it's integral corresponds to the electron charge q.

Exercise 5.2

A PIN photodiode is used with responsivity $\mathcal{R} = 1.25\,A/W$. The thermal noise for a reference bandwidth $B = 1/2T_b$ has a variance $\sigma_{th}^2 = 10^{-12}A^2$. The required quality parameter is $Q = 6$.

1) Find the required power P (receiver sensitivity).
2) Find the required α value for a 10 dB improvement with respect the benchmark case $\alpha = 1$.
3) Find the optimum α value and the corresponding sensitivity. Compare it with the benchmark case.

Exercise 5.3

A PIN-based receiver with thermal noise spectral density $S_{th} = 2 \cdot 10^{-22}\, A^2/Hz$ is used to detect an ideal intensity modulated signal at a bit rate $R_b = 10\, Gb/s$ and with a quality requirement $Q = 6$. Assume receiver's impulse response $h(t)$ to be ideal.

1) Find the maximum dark current if the sensitivity penalty should be limited to 1 dB.

Assume now an equivalent APD-based receiver (same thermal noise) with a noise factor $F = M^{1/2}$ where M is the photodiode's gain parameter. The primary dark current in an APD is amplified by the gain $I_D(M) = M \cdot I_D(0)$.

2) Find the maximum (primary) dark current if the sensitivity penalty should be limited to 1 dB when the gain is set to $M = 60$.
3) A typical primary dark current is $I_D = 50\, nA$. Find the corresponding sensitivities (average received power) for the PIN-based receiver and the APD-based receiver considering a responsivity $R = 0.8\ A/W$.

Exercise 5.4

A transmitter emits a maximum optical power $P = 1\,mW$ at a wavelength $\lambda = 1.55\,\mu m$. Such transmitter generates an ideal intensity modulated signal at a bit rate $R_b = 10\,Gb/s$. The optical link consists of a standard single-mode fiber (SSMF) with attenuation $\alpha = 0.2\,dB/km$ at the reference wavelength. Find the maximum transmission distance limited by attenuation for a bit error probability $BER = 10^{-9}$ in the following situations:

1) When the receiver is completely ideal (quantum limit).
2) For a PIN-based receiver with negligible dark current and ideal quantum efficiency knowing that the thermal noise spectral density is $S_{th} = 2 \cdot 10^{-22}\,A^2/Hz$. Assume receiver's impulse response $h(t)$ to be ideal.
3) For an optimized APD-based analogous receiver (same dark current, thermal noise and impulse response).
4) Same receiver as in question 3) including an optical preamplifier ($G = 40\,dB, NF = 5\,dB$) plus an ideal optical filter ($B_O = 100\,GHz$).

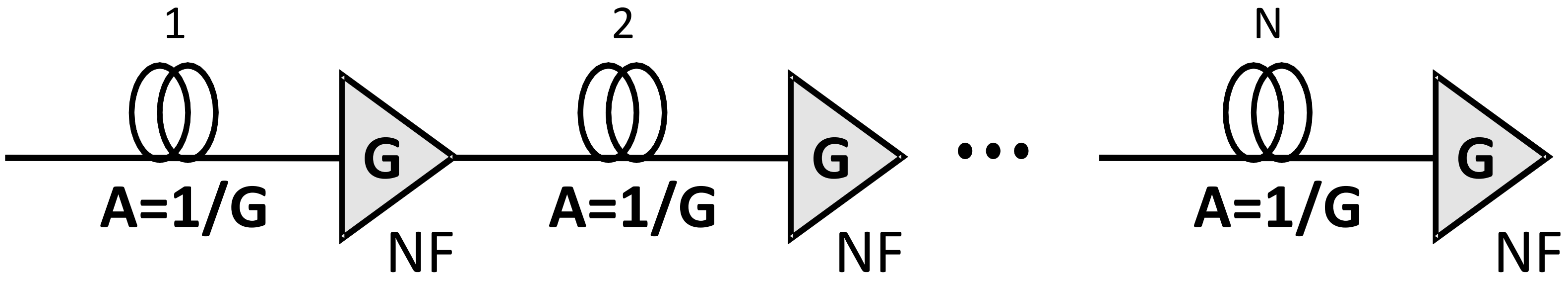
Exercise 5.5

A PIN-based receiver with thermal noise spectral density $S_{th} = 2 \cdot 10^{-22}\, A^2/Hz$ is used to detect an ideal intensity modulated optical carrier at a wavelength $\lambda = 1.55\, \mu m$ with a bit rate $R_b = 10\, Gb/s$. An EDFA with gain $G = 20\, dB$ and noise factor $NF = 5\, dB$ is used as a preamplifier. Assume both the electrical and optical filters to be ideal. The quality requirement is $Q = 6$.

1) Find the receiver's sensitivity (average power).
2) Find the penalty imposed by a finite extinction ratio $ER = 10\, dB$.
3) Find the required ER if the penalty should be limited to $1\, dB$.

Exercise 5.6

Consider a link composed by identic spans of fiber plus amplifier as shown in the figure. The span length is 100 km and the attenuation parameter at the reference wavelength $\lambda = 1550\, nm$ is $\alpha = 0.2\ dB/km$. The amplifiers' gain must perfectly compensate for the losses introduced by the previous fiber. The noise factor common to all amplifiers is $NF = 5\, dB$.



1) Find the OSNR after 10 spans (1000 km) if the transmitted power is $P_{tx} = 1\, mW$ and the bit rate is $R_b = 10\, Gb/s$.
2) Assuming a PIN-based receiver with ideal optical and electrical filters, find the maximum number of spans if a bit error ratio of $BER \leq 10^{-9}$ is required.
3) Find the optimum span length to maximize the transmission distance.